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PHYSICS OF THE MESOSCALE ATMOSPHERIC TURBULENCE: LABORATORY EXPERIMENTS

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Summary New experimental results in two-dimensional turbulence shed light on the origin of the Nastrom-Gage spectrum of kinetic energy and of the energy flux in the mesoscale atmospheric turbulence. It is shown that the large-scale flows strongly modify statistical properties of turbulence leading to apparent deviations from the Kolmogorov-Kraichnan theory. The isotropic turbulence result is revealed by the mean flow subtraction, a technique which should also be useful in the analysis of atmospheric data.

Introduction

Dual cascades of energy and enstrophy predicted by Kraichnan in 2D turbulence [1] lead to the spectral power of $E(k) = C\epsilon^{2/3}k^{-5/3}$ for the wave numbers larger than the wave number k_f of the forcing scale, $k < k_f$(the inverse energy cascade range), and to $E(k) = C_q\eta^{2/3}k^{-3}$ for the small scales, $k < k_f$ (forward enstrophy cascade range). Here ϵ and η are the dissipation rates of energy and enstrophy correspondingly. These two ranges have been found in numerical simulations of the 2D turbulence and have also been confirmed in experiments. Another important result in the theory of 2D turbulence is the Kolmogorov four-fifth law for the third-order velocity structure function [2], $S_3 = \left\langle (\delta V_L)^3 \right\rangle + \left\langle \delta V_L \left(\delta V_T\right)^2 \right\rangle = 2\epsilon r$. Here δV denotes the difference of velocities at two points separated by distance r. Angular brackets denote ensemble averaging over realizations, and the subscripts denote the longitudinal ($_L$) and transverse ($_T$) velocity components relative to *r*. Positive S_3 corresponds to the inverse energy cascade from small to large scales.

Motions of the atmosphere at scales exceeding its depth (10 km for troposphere) must be close to two-dimensional (2D), and the energy cascade is expected to be inverse, i.e. driven by a small-scale source. Yet atmospheric measurements seem to contradict the theory of 2D turbulence. First, the k^{-3} spectral range is observed at large scales (1000-3000 km), while the $k^{-5/3}$ range is observed in the mesoscale turbulence [3], i.e. in the reversed order with regard to 2D turbulence expectations. Second, the third-order velocity moment, which is related to the energy flux in homogeneous turbulence, is negative which suggests a direct energy cascade [4].

We propose a solution to this controversy by accounting for a presence of coherent large-scale flows in 2D turbulence. Presented laboratory experiments show that in a bounded system the inverse energy cascade leads to the acummulation of spectral energy at the system scale and the formation of mean flow or a condensate. The presence of the condensate modifies the kinetic energy spectrum of turbulence as well as the third-order velocity moments making them very similar to those observed in atmosphere. It is also shown that in the presence of the condensate the inverse energy cascade remains an underlying mechanism of the spectral transfer.

Generation of spectral condensate

The experiments are performed in the stratified layers of fluid. Turbulence is generated in a cell electromagnetically by driving an electric current through the top (conducting) fluid layer in the spatially varying vertical magnetic field [5]. A forcing scale, $l_f \approx$10 mm, is determined by the size of, and the spacing between, the magnets, and it is much smaller than the size of the square boundaries, L=100-240 mm. The inverse cascade proceeds up to the integral scale $\lambda_E \approx \epsilon^{1/2}\alpha^{-3/2}$, where α is the linear damping rate. If $\lambda_E > L$, the energy accumulates at the scale L generating spectral condensate. Fig. 1 shows the energy spectrum of the quasi-2D turbulence measured in the large boundary (L = 0.235 m) and intermediate damping α = 0.16 s^{-1}. A peak at $k_f \approx 400m^{-1}$ corresponds to the forcing scale. At $k > k_f$, the spectrum scales as $E(k)\ k^{-3}$, while at $k < k_f$ it scales close to . At $k > 80m^{-1}$, in the spectral condensate range, the spectrum is steeper and close to $E(k)\ k^{-3}$. Due to the presence of the condensate this spectrum, similarly to the Nastrom-Gage spectrum [3], has the k^{-3} and the $k^{-5/3}$ ranges in the reversed order for the large and intermediate scales.

The wave number at which the two power laws (the condensate k^{-3} and the inverse energy cascade $k^{-5/3}$) meet, denoted as $k_t = \pi/l_t$ in Fig. 1, can be estimated from the dimensional reasoning. It can be shown that the knee of the spectrum at the scale l_t is given by $k_t = \pi L^{-3/2}\left(C\alpha/2\right)^{-3/4}\epsilon^{1/4}$. The functional dependence of k_t on α and L has been tested experimentally resulting in a good qualitative agreement.

Energy flux in condensed turbulence

The spectral condensation can be viewed as the generation of mean flow or the large-scale coherent structures. Such structures can be revealed by a temporal averaging of the instantaneous velocity fields: $\overline{V} = 1/N \sum V(x,y)$. in the presence of the condensate, the velocity field contains both the mean component (which does not disappear as a result of the ensemble averaging) and turbulent velocity fluctuations, $V = \overline{V} + \widetilde{V}$. The statistical moments of the velocity increments are then modified as follows: $\delta V = \delta\overline{V} + \delta\widetilde{V}$, $S_2 = \left\langle (\delta V)^2 \right\rangle = \left\langle \left(\delta\overline{V}\right)^2 + \left(\delta\widetilde{V}\right)^2 + 2\delta\overline{V}\delta\widetilde{V} \right\rangle$, $S_3 =$

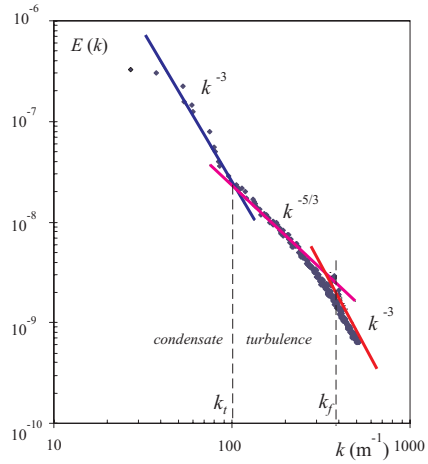

Figure 1. Kinetic energy spectrum of quasi-2D turbulence in the presence of spectral condensate. The turbulence range is defined at $k > k_t$, while the condensate range is $k < k_t$. The guide lines show the power laws for different spectral ranges: the k^{-3} enstrophy cascade, the $k^{-5/3}$ energy cascade, and the k^{-3} condensate range.

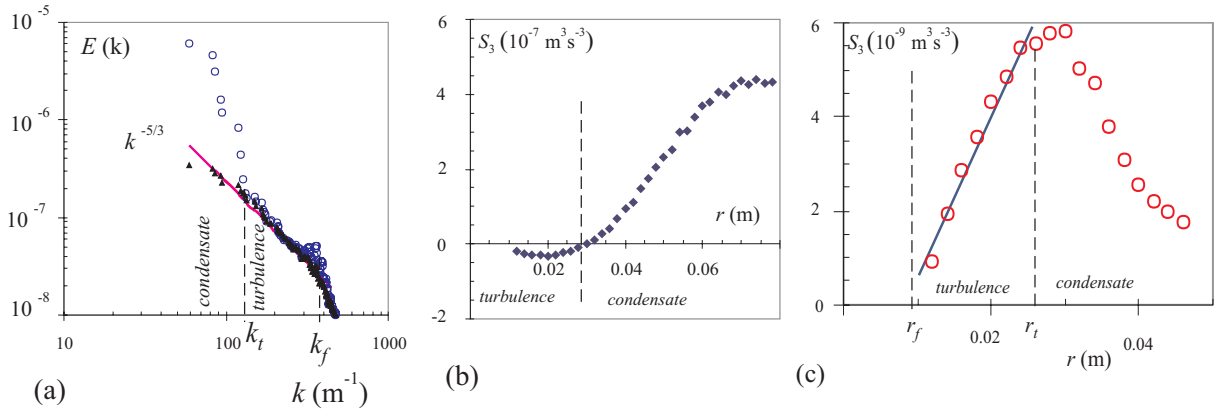

Figure 2. (a) Kinetic energy spectra before (circles) and after (triangles) mean subtraction. The third-order velocity moments (b) before and (c) after mean subtraction.

$\left\langle (\delta V)^3 \right\rangle = \left\langle \left(\delta\overline{V}\right)^3 + \left(\delta\widetilde{V}\right)^3 + 3\left(\delta\widetilde{V}\right)^2 \delta\overline{V} + 3\left(\delta\overline{V}\right)^2 \delta\widetilde{V} \right\rangle$, etc. The structure functions computed from the total velocity fields differ substantially from those expected from the Kolmogorov theory based on $\delta V = \delta\,\widetilde{V}$. To recover the Kolmogorov result one needs to subtract mean (time-average) velocity field from the instantaneous velocity fields. Fig. 2 shows the effect of the mean subtraction on the kinetic energy spectra and the third-order moments of velocity. The mean subtraction recovers Kolmogorov-Kraichnan spectrum (Fig. 2a). The third moment S_3 before subtracting mean (Fig. 2b) is negative in the turbulence range of scales, very similar to the atmosperic result [4]. After mean subtraction, a positive S_3 which changes linearly with the separation scale r is recovered, as seen in Fig. 2c. Positive S_3 is indicative of the inverse energy cascade. These results suggest that a similar analysis applied to the atmospheric wind data may help revitalizing the hypothesis about the inverse energy cascade in the mesoscale turbulence in the earth atmosphere.

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